Range Only Multistatic Tracking *

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Abstract – *Multistatic tracking involves using non-collocated transmitters and receivers to track the targets. In this paper we also assume no angle information. This paper proposes an approach using Gaussian Mixture representation of measurement uncertainty. An arbitrary number of receivers can be used using this approach. Here we limit ourselves to the track estimation issues, i.e. we assume no clutter measurement, and imperfect target detection at the receivers. Simulation results are presented which vindicate this approach.*

Keywords: Nonlinear estimation, target tracking, non-linear measurement fusion, multistatic tracking, range only.

1 Introduction

Multistatic tracking involves using non-collocated transmitters and receivers to track the targets. This introduces many benefits, and is an active area of research, a nice overview can be found in [1].

In this paper we also assume no angle information, which introduces a severe measurement non-linearity and also necessitates using more than one receiver (sensor) to ensure system observability. This paper investigates target position estimation and measurement fusion under such circumstances. Observability of related problem of tracking using monostatic range only measurements is investigated and analyzed in [2]. No clutter is assumed, and the probability of target detection is less than unity; in other words at each measurement time variable number of sensors will report measurements. This paper proposes an approach using Gaussian Mixture representation of measurement likelihood, which is then processed by a track splitting filter, the approach initially proposed in [3], and implemented for various non-linear estimation problems in [4–6]. One benefit of this approach is identical track update whether we receive one or arbitrary number of simultaneous receiver measurements.

Simulation results are presented which vindicate this approach.

Section 2 presents the problem statement and models and assumptions used in this paper. Approximation of measurement likelihood by a Gaussian Mixture in the Cartesian surveillance space is derived in Section 3. This approximation is used in target tracking updates, specified in Section 4. This approach is validated by a simulation study, presented in Section 5, followed by concluding remarks in Section 6.

A note on notation: Matrices are denoted by bold upper case letters, vectors are denoted by bold lower case letters or symbols, and scalars are denoted by standard (upper or lower case) letters or symbols.

2 Problem Statement

In this paper we consider two dimensional multistatic surveillance scenarios. A single emitter broadcasts signal, which is received by multiple sensors. To simplify notation, we consider stationary emitter and sensors, although the extension to mobile emitters and sensors is straightforward and minimal.

Each sensor receives both the direct signal from emitter, and the signal from emitter and reflected by the target. The time difference between these signals is a measurement of the difference in two lengths the signal had to travel. Here we consider target tracking using the path difference measurements provided by multiple sensors.

The sensors have uncertain detection, thus the measurement may come from zero or more sensors. Proposed algorithm handles a variable number of measurements arriving from multiple sensors simultaneously, with no upper limit on the number of measurements (assuming, of course, sufficient computational resources).

We consider a single target situation here, with no clutter. Thus there are no data association issues, as the origin of each measurement is known.

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2.1 Target Trajectory

Denote by $\mathbf{x}_k$ target trajectory state at time k. Assume that the target trajectory state propagation from time $k-1$ to time k is modeled by

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \boldsymbol{\omega}_k \tag{1}$$

with $\boldsymbol{\omega}_k$ a sample of zero mean, white Gaussian process with covariance $\mathbf{Q}$.

Target trajectory is usually expressed in Cartesian coordinates, almost always with position and velocity as elements of the trajectory state. Here we assume that exists linear projection $\mathbf{H}$ from the trajectory state space into the surveillance (position) space, i.e. that $\mathbf{H}\mathbf{x}_k$ equals the position of the target.

2.2 Sensors

Multiple sensors receive emitter signals and deliver the "distance difference" measurements. Here we assume that sensors do not provide any angular information, although the algorithm accommodates angular information in a straightforward way indeed.

To ensure observability under these assumptions, we need multiple (at least two) sensors, which must be non-collocated and non-collinear with the transmitter (the second condition includes the first).

Imperfect detection is assumed, thus a variable (non-negative) number of measurements are received at each measurement time.

2.3 Measurements

Each sensor corrupts measurement signal with additive measurement noise, and the measurement from sensor (s) at time k equals

$$y_k^{(s)} = h\left(\mathbf{x}_k; \mathbf{x}^{(s)}\right) + \nu_k^{(s)} \tag{2}$$

with $\mathbf{x}^{(s)}$ denoting sensor trajectory state, and $\nu_k^{(s)}$ denoting a sample of zero mean, white Gaussian process with standard deviation $\sigma^{(s)}$ uncorrelated with process $\boldsymbol{\omega}$ and any sequence $\nu_k^{(t)}$, $(s) \neq (t)$. Nonlinear function h is detailed below.

2.3.1 Uncertainty Ellipse

A measurement situation of sensor (1) is depicted in Figure 1. Denote by $\mathbf{d}_k^{(s)}$ and by $d_k^{(s)}$ the distance vector and distance from sensor (s) to the target respectively

$$\mathbf{d}_k^{(s)} = \mathbf{H}\left(\mathbf{x}_k - \mathbf{x}^{(s)}\right) \tag{3}$$

$$d_k^{(s)} = \left\|\mathbf{d}_k^{(s)}\right\|, \tag{4}$$

and denote by $\mathbf{e}_k$ and by e_k the distance vector and distance from the emitter (transmitter $\mathbf{t}$) to the target respectively

$$\mathbf{e}_k = \mathbf{H}\left(\mathbf{x}_k - \mathbf{t}\right) \tag{5}$$

$$e_k = \left\|\mathbf{e}_k\right\|. \tag{6}$$

Denote by $a_k^{(s)}$ the total distance from the emitter to target to sensor (s)

$$a_k^{(s)} = d_k^{(s)} + e_k \tag{7}$$

and by $b^{(s)}$ the direct distance from sensor (s) to the emitter

$$\mathbf{b}^{(s)} = \mathbf{H}\left(\mathbf{t} - \mathbf{x}^{(s)}\right) \tag{8}$$

$$b^{(s)} = \left\|\mathbf{b}^{(s)}\right\| \tag{9}$$

Measurement function from equation (2) is

$$h\left(\mathbf{x}_k; \mathbf{x}^{(s)}\right) = a_k^{(s)} - b^{(s)} \tag{10}$$

which is, of course, nonlinear with respect to $\mathbf{x}_k$. For simplification, we assume that the positions of the emitter and the sensors are known. In this paper we will also use words "transmitter" and "receiver" for the "emitter" and "sensor" respectively.

To determine measurement likelihood, assume initially that we have a single sensor (s). First observe that, given the value of $h(\mathbf{x}_k; \mathbf{x}^{(s)})$, and therefore the value $a_k^{(s)}$, possible target positions form an ellipse where the emitter and sensor (s) form the focal points, and with the major axis of $a_k^{(s)}$, and the minor axis of

$$\sqrt{\left(a_k^{(s)}\right)^2 - \left(b^{(s)}\right)^2}. \tag{11}$$

If the target was anywhere on this ellipsis, the measurement equation (10) would yield the same value.

2.3.2 Parametrized Uncertainty Ellipse

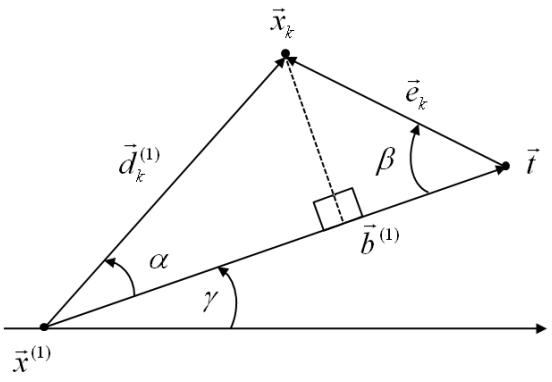

Figure 1: Sensor (1) measurement geometry

For reasons which will be clear below, we are interested in a parametrized version of uncertainty ellipse. Denote by γ the

angle of unit vector from sensor (s) to the emitter (baseline (s)),

$$\mathbf{i}_\gamma = \begin{bmatrix}\cos(\gamma)\\ \sin(\gamma)\end{bmatrix} = \frac{\mathbf{b}^{(s)}}{b^{(s)}}, \tag{12}$$

denote by α the angle centered at sensor (s) from baseline (s) to the target (or from baseline (s) to $\mathbf{d}_k^{(s)}$), and denote by β the angle centered at the emitter, from baseline (s) to the target (or from baseline (s) to $\mathbf{e}_k$. Angles α and β are inside angles formed by triangle of baseline (s), $\mathbf{d}_k^{(s)}$ and $\mathbf{e}_k$. Referring to Figure 1, it is easy to see that the following trigonometric relations hold:

$$d_k^{(s)}\ \cos(\alpha) + e_k\ \cos(\beta) = b^{(s)} \tag{13}$$

$$d_k^{(s)}\ \sin(\alpha) = e_k\ \sin(\beta) \tag{14}$$

$$d_k^{(s)} + e_k = a_k^{(s)} \tag{15}$$

Solving it we obtain

$$d_k^{(s)}\left(\alpha; a_k^{(s)}\right) = \frac{\left(a_k^{(s)} + b^{(s)}\right)\left(a_k^{(s)} - b^{(s)}\right)}{2\left(a_k^{(s)} - b^{(s)}\ \cos(\alpha)\right)} \tag{16}$$

where α has the role of parameter, $b^{(s)}$ is assumed known, and $a_k^{(s)}$ is given. Vector $\mathbf{d}_k^{(s)}$ equals (Figure 1)

$$\begin{aligned}\mathbf{d}_k^{(s)}\left(\alpha; a_k^{(s)}\right) &= d_k^{(s)}\left(\alpha; a_k^{(s)}\right)\mathbf{i}_{\gamma+\alpha}\\ &= d_k^{(s)}\left(\alpha; a_k^{(s)}\right)\begin{bmatrix}\cos(\gamma+\alpha)\\ \sin(\gamma+\alpha)\end{bmatrix}\end{aligned} \tag{17}$$

The target position, assuming angle α and value of $a_k^{(s)}$ equals

$$\begin{aligned}\mathbf{x}_k\left(\alpha; a_k^{(s)}\right) &= \mathbf{f}_k^{(s)}\left(\alpha; a_k^{(s)}\right)\\ &\triangleq \mathbf{x}^{(s)} + \mathbf{d}_k^{(s)}\left(\alpha; a_k^{(s)}\right).\end{aligned} \tag{18}$$

The uncertainty ellipse can be formed by incrementing α along its range and calculating $\mathbf{x}_k\left(\alpha; a_k^{(s)}\right)$ from (18).

2.3.3 Observability Issues

It is self evident that the target is not observable when using a single sensor; i.e. there is an infinite number of possible target locations which will produce the measurements. That is true for both stationary and mobile targets. In this paper we assume two sensors, with the understanding that it is straightforward to extend the results to more than two sensors.

Due to the probability of detection being less than one, sometimes the measurement from only one sensor will be available for track update, and sometimes both measurements will be present. Proposed algorithm is able to use all available measurements, whether they arrive singly or in pairs (or, given more sensors, in more numbers simultaneously).

Tracks may be initialized by either a single measurement or by more than one simultaneous measurements.

3 Measurement Likelihood Gaussian Mixture Presentation

Due to the noise in measurements obtained (2) the possible target locations no longer form an ellipse, rather they become an area around the ellipse defined in Section 2.3. This is definitively not a Gaussian likelihood required for linear estimation.

Denote by $\mathbf{y}_k$ the set of measurements received by sensors at time k. In this case we consider receiving measurement from one or from two sensors, with further extension self evident.

In this paper we propose using GMM-ITS algorithm for tracking targets in this environment, which requires approximating the measurement likelihood by a Gaussian mixture [3–6] in surveillance space

$$p(\mathbf{y}_k|\mathbf{x}_k) \approx c_y \sum_{g=1}^{G} \gamma_g \mathcal{N}\left(\mathbf{z}_{k,g}; \mathbf{H}\mathbf{x}_k, \mathbf{R}_{k,g}\right) \tag{19}$$

where $\mathcal{N}(\mathbf{x}; \mathbf{m}, \mathbf{P})$ denotes the Gaussian pdf of variable $\mathbf{x}$ with mean $\mathbf{m}$ and covariance $\mathbf{P}$. The value of constant c_y is irrelevant as it cancels out in the Bayes equation. For simplicity we make $c_y = 1$ and ignore in the rest of the paper. Treating the Gaussian Mixture components as mutually exclusive and exhaustive events, we obtain

$$\sum_{g=1}^{G} \gamma_g = 1, \quad \gamma_g > 0 \tag{20}$$

constraint. In other words, nonlinear measurement(s) $\mathbf{y}_k$ are replaced by a set of G triplets $\{\gamma_g, \mathbf{z}_{k,g}, \mathbf{R}_{k,g}\}$, where $\mathbf{z}_{k,g}$, $\mathbf{R}_{k,g}$ represent mean value and covariance of a linear measurement in surveillance space, and γ_g is the relative probability that "linear measurement" g is correct.

3.1 Single Measurement

The procedure to approximate measurement likelihood of a single multistatic measurement follows closely the procedure described in [4–6], albeit for different types of measurements.

One sigma range of bistatic measurement by sensor (s) equals $y_k^{(s)} \pm \sigma^{(s)}$. This range projects into the surveillance space bounded by curves $\mathbf{x}_k\left(\alpha; y_k^{(s)} + b^{(s)} \pm \sigma^{(s)}\right)$ of (18). This area can be divided into ellipsoids, each of which is a one-σ footprint of a Gaussian Mixture component. This result is shown in Figure 2 where the uncertainty area of the measurement received by "receiver 1" resulted in the red

coloured string of ellipsoids themselves forming an ellipse with the emitter and receiver 1 serving as the focal points.

The curves are segmented into a relatively small number of segments, thus we only need values of potential target locations $\mathbf{x}_k(\alpha;\cdot)$, equation (18) at a limited set of angles denoted here by α_i, $i = 1,\ldots,G+1$. Value of G is a trade off between computational resources and performance.

To obtain segment g, we use angles α_g and α_{g+1}, which intersect the two curves at four points

$$\begin{bmatrix}\mathbf{q}_1\\ \mathbf{q}_2\\ \mathbf{q}_3\\ \mathbf{q}_4\end{bmatrix} = \begin{bmatrix}\mathbf{f}_k^{(s)}(\alpha_g; & y_k^{(s)}+b^{(s)}+\sigma^{(s)})\\ \mathbf{f}_k^{(s)}(\alpha_{g+1}; & y_k^{(s)}+b^{(s)}+\sigma^{(s)})\\ \mathbf{f}_k^{(s)}(\alpha_g; & y_k^{(s)}+b^{(s)}-\sigma^{(s)})\\ \mathbf{f}_k^{(s)}(\alpha_{g+1}; & y_k^{(s)}+b^{(s)}-\sigma^{(s)})\end{bmatrix}. \quad (21)$$

One semi axis of the inscribed ellipsoid is bounded by

$$\begin{aligned}\mathbf{q}_{c1} =&(\mathbf{q}_1+\mathbf{q}_3)/2\\ \mathbf{q}_{c2} =&(\mathbf{q}_2+\mathbf{q}_4)/2,\end{aligned} \quad (22)$$

and has length D_c and unit vector $\mathbf{i}(\alpha_c)$ of

$$\begin{aligned}\Delta\mathbf{q}_c =&\mathbf{q}_{c1}-\mathbf{q}_{c2}\\ D_c =&\|\Delta\mathbf{q}_c\|/2\\ \mathbf{i}(\alpha_c) \triangleq& \begin{bmatrix}\cos(\alpha_c)\\ \sin(\alpha_c)\end{bmatrix} = \frac{\Delta\mathbf{q}_c}{\|\Delta\mathbf{q}_c\|}.\end{aligned} \quad (23)$$

The length D_o of the other semi axis is given by

$$D_o = \begin{bmatrix}-\sin(\alpha_c)\\ \cos(\alpha_c)\end{bmatrix}^T \frac{1}{2}\left(\frac{\mathbf{q}_1-\mathbf{q}_3}{2}+\frac{\mathbf{q}_2-\mathbf{q}_4}{2}\right) \quad (24)$$

$$= \mathbf{i}(\alpha_c+\pi/2)^T\left(\frac{\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3-\mathbf{q}_4}{4}\right). \quad (25)$$

The center of the inscribed ellipsoid is given by

$$\mathbf{z}_{k,g} = 0.5(\mathbf{q}_{c1}+\mathbf{q}_{c2}), \quad (26)$$

which is also the mean of the measurement component g corresponding to the ellipse. Denote α rotation matrix by $\mathbf{T}(\alpha) = \begin{bmatrix}\mathbf{i}(\alpha) & \mathbf{i}(\alpha+\pi/2)\end{bmatrix}$. The covariance matrix of measurement component g is

$$\mathbf{R}_{k,g} = \mathbf{T}(\alpha_c)\begin{bmatrix}D_c^2 & 0\\ 0 & D_o^2\end{bmatrix}\mathbf{T}(\alpha_c)^T. \quad (27)$$

Finally, the probability that the target is within the footprint of a measurement component is proportional to the area of the footprint:

$$\gamma_g \propto \sqrt{|\mathbf{R}_{k,g}|}. \quad (28)$$

3.2 Multiple Measurements

If measurements from multiple sensors are simultaneously present, we have a choice. We can update the track by each of the measurements sequentially, or we can integrate measurement information and update the track with the joint likelihood. For the multistatic problem under investigation, the second approach is almost always computationally better.

The procedure is illustrated for the case of measurements from two sensors, however the procedure can be recursively extended to any number of simultaneous measurements.

Using procedure described in 3.1, we present the likelihoods of measurements $y_k^{(s)}$, $(s) = (1),(2)$ by a GM, as per (19) by

$$p(y_k^{(s)}|\mathbf{x}_k) \approx \sum_{g(s)=1}^{G(s)} \gamma_{g(s)}\mathcal{N}\left(\mathbf{z}_{k,g(s)}^{(s)};\mathbf{H}\mathbf{x}_k,\mathbf{R}_{k,g(s)}^{(s)}\right) \quad (29)$$

We next replace two existing GMs by a third ("new") GM. Elements of the new GM are denoted as in equation (19). Each pair of "old" components $g(1)$ and $g(2)$ forms one new component g. This may be computationally expensive, however the computational burden may be reduced significantly by employing measurement selection strategies of target tracking arsenal [7–9]. The details of this selection process are omitted for reasons of clarity, to concentrate on important points of this paper.

The "new" GM is parameterized by

$$\begin{aligned}\gamma_g \propto& \gamma_{g(1)}\gamma_{g(2)} \quad (30)\\ &\mathcal{N}\left(\mathbf{z}_{k,g(1)}^{(1)};\mathbf{z}_{k,g(2)}^{(2)},\mathbf{R}_{k,g(1)}^{(1)}+\mathbf{R}_{k,g(2)}^{(2)}\right)\end{aligned}$$

$$(\mathbf{R}_{k,g})^{-1} = \left(\mathbf{R}_{k,g(1)}^{(1)}\right)^{-1}+\left(\mathbf{R}_{k,g(2)}^{(2)}\right)^{-1} \quad (31)$$

$$\begin{aligned}\mathbf{z}_{k,g} =& \mathbf{R}_{k,g} \quad (32)\\ &\left(\left(\mathbf{R}_{k,g(1)}^{(1)}\right)^{-1}\mathbf{z}_{k,g(1)}^{(1)}+\left(\mathbf{R}_{k,g(2)}^{(2)}\right)^{-1}\mathbf{z}_{k,g(2)}^{(2)}\right)\end{aligned}$$

where γ_g is normalized to satisfy (20).

4 Target Tracking

In this paper we assume no uncertainty of target existence and of origin of measurement(s). In these circumstance, target tracking equates to the (recursive) target trajectory state estimation.

Target trajectory state estimation proposed for this problem is GMM-ITS, proposed in [3] and also used in [4–6].

We use $\mathbf{Y}^k$ to denote the sequence of all sets of sensor measurements $\mathbf{y}_\ell$, $\ell = 1,\ldots,k$.

4.1 Track State

Track state pdf is initialized and updated by nonlinear measurement(s). Therefore, the state pdf is non-Gaussian, and is approximated here by a Gaussian Mixture. Each component of the GM presentation of the track state pdf is termed a track component (as opposed to the measurement components of measurement likelihood GM presentation). This is the track state pdf approximation after measurements of time $k-1$ have been processed:

$$p\left(\mathbf{x}_{k-1}|\mathbf{Y}^{k-1}\right) = \sum_{c=1}^{C_k} \xi_c \mathcal{N}\left(\mathbf{x}_{k-1}; \hat{\mathbf{x}}_{k-1|k-1}^{(c)}, \mathbf{P}_{k-1|k-1}^{(c)}\right) \tag{33}$$

where c denotes track component, or index of the track component, depending on the context, and

$$\sum_{c=1}^{C_k} \xi_c = 1, \quad \xi_c > 0. \tag{34}$$

The advantage of using GMs for both measurement likelihood presentation and for the track state pdf presentation, is that we may replace non-linear operations by a number of simple Kalman filter updates.

4.2 Track Recursion

Track recursion at time k starts with the track state pdf obtained after the update of measurement(s) obtained at time $k-1$, equation (33), and updated by measurement likelihood (19).

Here we briefly list the elements of one track recursion cycle.

- track prediction,
- measurement component selection and likelihood calculation,
- track update,
- track component management, and
- track output.

We start the track cycle at the moment of arrival of new measurement(s) at time k. The elements of the cycle are similar with other ITS based implementations [6, 10].

4.2.1 Track Prediction

Track prediction propagates track state pdf from time $k-1$ to time k. This boils down to propagating each track component individually and

$$p\left(\mathbf{x}_k|\mathbf{Y}^{k-1}\right) = \sum_{c=1}^{C_k} \xi_c \mathcal{N}\left(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k-1}^{(c)}, \mathbf{P}_{k|k-1}^{(c)}\right) \tag{35}$$

Relative probability ξ_c of each component c does not change when propagating. As we assume a single trajectory model, the mean and covariance of each component propagate as per standard Kalman filter prediction denoted by $\mathrm{KF_P}$

$$\left[\hat{\mathbf{x}}_{k|k-1}^{(c)}, \mathbf{P}_{k|k-1}^{(c)}\right] = \mathrm{KF_P}\left(\hat{\mathbf{x}}_{k-1|k-1}^{(c)}, \mathbf{P}_{k-1|k-1}^{(c)}, \mathbf{F}, \mathbf{Q}\right). \tag{36}$$

It is straightforward to include IMM propagation [11] for the case of maneuvering target tracking. The same is valid for all elements of the track recursion cycle.

4.2.2 Selection and Likelihood Calculation

In theory, each track component should get updated by each measurement component. In practice it will often result in a waste of computational resources. Each track component c will select a subset of measurement components for update. The selection procedure is identical to track selection procedure described in many other references [7–9], and will not be detailed here. Briefly, each component selects measurement components with significant mutual likelihoods.

The likelihood of a selected measurement component g, with respect to the track component c is

$$p_{k,g}^{(c)} = \mathcal{N}\left(\mathbf{z}_{k,g}; \mathbf{H}\hat{\mathbf{x}}_{k|k-1}^{(c)}, \mathbf{S}_{k,g}^{(c)}\right) \tag{37}$$

where

$$\mathbf{S}_{k,g}^{(c)} = \mathbf{H}\mathbf{P}_{k|k-1}^{(c)}\mathbf{H}^T + \mathbf{R}_{k,g}. \tag{38}$$

The likelihood of a measurement component g which is not selected by the track component c, and with respect to the track component c is $p_{k,g}^{(c)} = 0$.

4.2.3 Track Update

During the track update, each existing track component is replaced by a number of new track components. Each pair of track component c and a measurement component g form a new component, given that track component c selects measurement component g. Denote by c^+ new component formed by track component c and measurement component g, then the updated track state pdf is approximated by

$$p\left(\mathbf{x}_k|\mathbf{Y}^k\right) = \sum_{c^+=1}^{C_{k+1}} \xi_{c^+} \mathcal{N}\left(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k}^{(c^+)}, \mathbf{P}_{k|k}^{(c^+)}\right) \tag{39}$$

where relative component probability ξ_{c^+} is

$$\xi_{c^+} \propto \xi_c \gamma_g p_{k,g}^{(c)}. \tag{40}$$

which has to be normalized to satisfy constraint

$$\sum_{c^+}^{C_{k+1}} \xi_{c^+} = 1. \tag{41}$$

Mean $\hat{\mathbf{x}}_{k|k}^{(c^+)}$ and covariance $\mathbf{P}_{k|k}^{(c^+)}$ of new component c^+ are obtained by simple Kalman filter update of track component c by measurement component g

$$\left[\hat{\mathbf{x}}_{k|k}^{(c^+)}, \mathbf{P}_{k|k}^{(c^+)}\right] = \mathrm{KF}_{\mathrm{U}}\left(\hat{\mathbf{x}}_{k|k-1}^{(c)}, \mathbf{P}_{k|k-1}^{(c)}, \mathbf{z}_{k,g}, \mathbf{R}_{k,g}, \mathbf{H}\right), \tag{42}$$

where KF_{U} denotes the Kalman filter update operation.

4.2.4 Track Component Management

The number of track components grows exponentially in time, as each track component is, as a rule, replaced by more than one measurement component. This would quickly saturate available computational resources. Thus the proposed algorithm would not be practical without some track component management implementation [12]. Proposed algorithm is an instance of track splitting algorithm [3,10,13,14], and all track splitting track component management methods can also be used here.

Without going into details, they include track component pruning, where components with small relative probabilities ξ are removed, track component subtree pruning, where whole subtrees of components are removed based on track component relative probabilities [7]. Finally, one can merge "close" track components, where definition of "close" differs between various proposals [15–17].

The bottom line is that the number of components C_{k+1} is reduced to the level consistent with the available computational resources.

4.2.5 Track Output

There are multiple ways one can output state of the track splitting filter [7].

One possibility is to output track estimate mean $\hat{\mathbf{x}}_{k|k}$, and track estimate covariance $\mathbf{P}_{k|k}$, defined as the mean and covariance of *a posteriori* track state pdf at time k

$$\hat{\mathbf{x}}_{k|k} = \sum_{c^+} \xi_{c^+} \hat{\mathbf{x}}_{k|k}^{(c^+)} \tag{43}$$

$$\mathbf{P}_{k|k} = \sum_{c^+} \xi_{c^+} \left(\mathbf{P}_{k|k}^{(c^+)} + \hat{\mathbf{x}}_{k|k}^{(c^+)} \left(\hat{\mathbf{x}}_{k|k}^{(c^+)}\right)^T\right) - \hat{\mathbf{x}}_{k|k}\hat{\mathbf{x}}_{k|k}^T \tag{44}$$

The other possibility is to output mean and covariance of the track component with highest relative probability ξ_{c^+}. In our Simulations we have used the second approach.

4.3 Track Initialization

In the (usual) absence of prior information, tracks are initialized using measurements. Here we initialize tracks using the first measurement time GM measurement likelihood presentation.

Assume that track state includes target position and speed in Cartesian coordinates (surveillance space). Each measurement component g translates into one track component c, and $C_2 = G_1$. The track component position elements of mean and covariance are equal to the measurement component mean and covariance

$$\mathbf{H}\hat{\mathbf{x}}_{1|1}^{(c^+)} = \mathbf{z}_{1,g} \tag{45}$$

$$\mathbf{H}\mathbf{P}_{1|1}^{(c^+)}\mathbf{H}^T = \mathbf{R}_{1,g}. \tag{46}$$

The component velocity elements of the track component c^+ mean equals zero, and the component velocity elements of the track component c^+ covariance depends on the maximum target velocity.

5 Simulation Study

We consider a simple two dimensional surveillance scenario. One stationary emitter is at position $(0, 0)$km, One target follows a straight line uniform motion trajectory, from position $(3, -2)$ km, with the speed vector of $(5\sqrt{2}, 5\sqrt{2})$m/s as indicated in Figure 2.

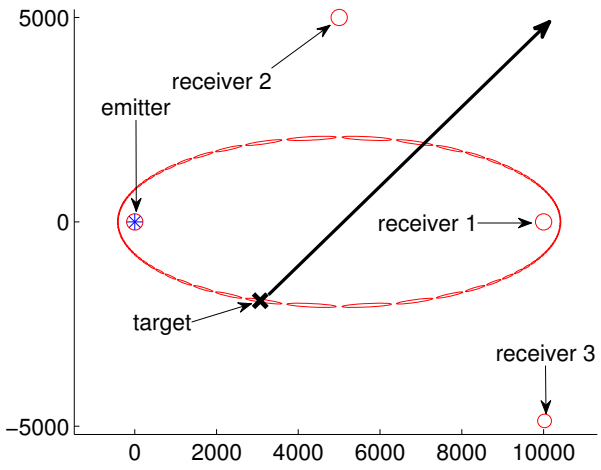

Figure 2: Simulation scenario.

There is no clutter, thus there are no data association issues. Probability of detection for each sensor equals $P_D = 1.0$.

Multistatic measurements are taken once in 10 seconds, and total simulated time is 1000 s. Each simulation scenario is repeated 500 times with independent measurements.

Two scenarios have been simulated. In one, two sensors labeled "receiver 1" and "receiver 2" on Figure 2 are used, positioned at $(10, 0)$ km and at $(5, 5)$ km respectively. In the other scenario the third sensor ("receiver 3") positioned at $(10, -5)$ km is also used. All the sensors are stationary and the measurement standard deviation is 20 m..

In each simulation run, the track is initialized based on the first received measurement(s). Track component management consists of merging track components with common history in last 2 scans, as well as pruning all track components with relative probability $\xi < 10^{-3}$.

In the two sensor scenario, the track diverges in 14.4% of the cases. Track divergence occurs as two measurements, one from each sensors, have measurements likelihoods as two ellipses. These two ellipses intersect at two points, one near the target, and the other near the "phantom" target. In a majority of cases, uniform target motion helps to increase the relative probabilities of track components in the vicinity of targets, but in these 14.4% cases the strongest track components start following the "phantom" target. The "weak" geometry is when the target is (approximately) on the straight line between the emitter and receiver 1, when the measurements from receiver 1 result in a relatively large measurement likelihood area.

This does not happen in the three sensor scenario, where the presence of the third measurement ensures that the track divergence does not happen. These measurements have most effects in the initial part of each simulation run, when the target is "near" receiver 3, whilst in the second half of each simulation run measurements from receiver 3 do not improve estimation process noticeably.

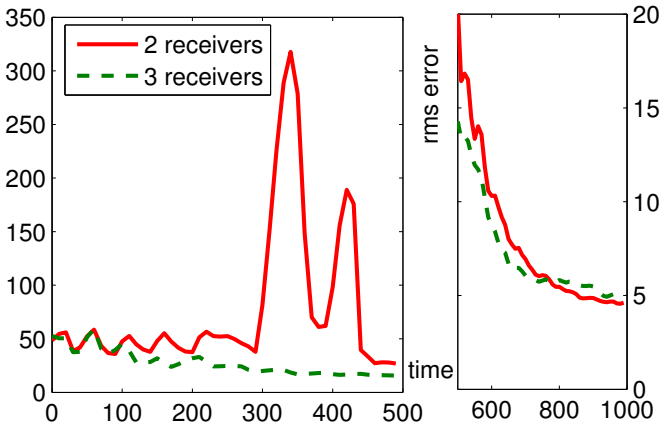

Figure 3: RMS estimation errors

This is illustrated in Figure 3 which depicts rms estimation errors. In the first half of the simulated time, the three sensor case has distinctive advantage in smaller estimation errors, whereas toward the end of the simulated time there is no clearly better outcome. After tracks diverge, they do not contribute to results presented in Figure 3.

Estimation results depend on the geometry; the contribution of additional sensors depend on the angle with which their measurement likelihood ellipse intersects corresponding ellipses from other sensors.

6 Conclusions

In this paper we present a way to estimate target trajectory (track the target) based on a number of simultaneous "range only" multistatic measurements. Here we consider a situation where target existence is not ambiguous, and there is no clutter. In other words, there are no data association issues. Target is detected by each sensor with a probability of detection, which is reflected in a variable number of multistatic measurements received at each scan.

The track is updated by a Gaussian Mixture of measurement components, regardless of how many simultaneous multistatic measurements are received.

Simulation results validate this approach. If some bearing information were available, they can be easily integrated with the proposed algorithm, the details are left for some future publication.

References

[1] S. Coraluppi, "Multistatic sonar localization," *IEEE Journ. Oceanic Engineering*, vol. 31, no. 4, October 2006.

[2] T. L. Song, "Observability of target tracking with range–only measurements," *IEEE Journ. Oceanic Engineering*, vol. 24, no. 3, pp. 383–387, July 1999.

[3] D. Mušicki and R. Evans, "Measurement Gaussian sum mixture target tracking," in *9th International Conference on Information Fusion, Fusion 2006*, Florence, Italy, July 2006.

[4] D. Mušicki, "Multi-target tracking using multiple passive bearings-only asynchronous sensors," *IEEE Trans. Aerospace and Electronic Systems*, vol. 44, no. 3, pp. 1151–1160, July 2008.

[5] ——, "Bearings only single-sensor target tracking using Gaussian mixtures," *Automatica*, vol. 45, no. 9, pp. 2088–2092, September 2009.

[6] D. Mušicki, R. Kaune, and W. Koch, "Mobile emitter geolocation using TDOA and FDOA measurements," *IEEE Trans. Signal Processing*, vol. 58, no. 3, pp. 1863–1874, Mar 2010.

[7] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*. Artech House, 1999.

[8] Y. Bar-Shalom and E. Tse, "Tracking in a cluttered environment with Probabilistic Data Association," *Automatica*, vol. 11, pp. 451–460, Sep 1975.

[9] Y. Bar-Shalom, K. Chang, and H. Blom, *Multitarget Multisensor Tracking*. Artech House, 1990, vol. 1, ch. Automatic track formation in clutter with a recursive algorithm, pp. 25–42.

[10] D. Mušicki, B. La Scala, and R. Evans, "The Integrated Track Splitting filter - efficient multi-scan single target tracking in clutter," *IEEE Trans. Aerospace Electronic Systems*, vol. 43, no. 4, pp. 1409–1425, October 2007.

[11] H. Blom and Y. Bar-Shalom, "The Interacting Multiple Model algorithm for systems with Markovian switching coefficients," *IEEE Trans. Automatic Control*, vol. 33, no. 8, pp. 780–783, Aug 1988.

[12] S. Blackman, *Multiple-target tracking with radar applications*. Artech House, 1986.

[13] T. Kurien, *Multitarget Multisensor Tracking*. Artech House, 1990, vol. 1, ch. Issues in the design of practical multitarget tracking algorithms, pp. 43–83.

[14] D. B. Reid, "An algorithm for tracking multiple targets," *IEEE Trans. Automatic Control*, vol. 24, no. 6, pp. 843–854, Jun 1979.

[15] R. A. Singer, R. G. Sea, and K. Housewright, "Derivation and evaluation of improved tracking filters for use in dense multi-target environments," *IEEE Trans. Information Theory*, vol. 20, no. 4, pp. 423–432, Jul 1974.

[16] D. J. Salmond, "Mixture reduction algorithms for target tracking in clutter," in *SPIE: Signal and Data Processing of Small Targets*, vol. 1305, Orlando, Florida, April 1990, pp. 434–445.

[17] J. L. Williams and P. S. Mayback, "Cost-function-based gaussian mixture reduction for target tracking," in *6th International Conference on Information Fusion, Fusion 2003*, Cairns, Queensland, Australia, July 2003, pp. 1047–1054.